UTTARAKHAND OPEN UNIVERSITY, HALDWANI (NAINITAL)
उत्तराखंड मुक्त विश्वविद्यालय, हल्द्वानी (नैनीताल)

M.A. /M.Sc. Mathematics
ASSIGNMENT-SECOND YEAR

Last Date of Submission: 15 May जमा करने की अन्तिम तिथिः 15 मई

Course Title: Analysis and Advanced Calculus **Course Code: MAT-506**

Year: 2013-14 **Maximum Marks : 40**

Section 'A'
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Section 'A' contains 08 short answer type questions of 5 marks each. Learners are required to answers 4 questions only. Answers of short answer-type questions must be restricted to 250 words approximately.

Briefly discuss the following:

1. The limit of a convergent sequence is unique.
2. Every compact subset of a normed linear space is complete.
3. If a normed space N is reflexive, show that N* is reflexive. Symbols have usual meaning.
4. Define
   i. Inner product space
   ii. Hilbert space and give an example
5. Prove that an orthonormal set in a Hilbert space is linearly independent.
6. If T is a linear operator on a Hilbert space H then T is unitary iff adjoint of T exists and T T*= T *T = I
7. Let f be a regulated function on a compact interval [a, b] of R into a Banach space X. then it each t ∈ [a, b], the function F: [a, b] → X, F(t)= $\int_a^t f$, t ∈ [a , b] is continuous.
8. Define Directional derivative.

   of f : V → Y is differentiable at $x \in V$ X and X and Y are Banach space over the same field and V be an open subset of X. show that

   Dv $f\ (x) = D\ f(x)v$, where $v \in V$ is a unit vector.

Section 'B'
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- **Section 'B' contains 04 long answer-type questions of 10 marks each. Learners are required to answers 02 questions only.**

1. Let X and Y be Banach space over the same field K and V be an open subset of X Let f:V→ Y be an n-times differentiable function on V. Then for each permutation p of n and each point $(x_{1,} x_2,\text{--------}, x_n) \in$ X and each $v \in V,$ $D^n f(v)(x_{p(1)}, x_{p(2)},\text{------}, x_{p(n)}) = D^n f(v)(x_{1,} x_2,\text{--------}, x_n)$.

2. If T is an operator on a Hilbert space H, then $(Tx, x) = 0 \;\forall\; x \in H$ iff T= 0.

3. If M be a closed linear subspace of a normed linear space N, X_0 be a point in N but not in M and d be the distance from $X_0$to M.
   then show that ∃ a functional F in N (whole space) s.t. F(M)=0, $F(X_0)=1$ and $\|F\| = \frac{1}{d}$.

4. If T be a bounded linear transformation of normed space N into normed space N^1, then show that the following norms are equivalent:
   i. $\|T\| = Sup\left\{\frac{\|T\ (x)\|}{\|x\|} : x \neq\ 0\right\} x \in N$
   ii. $\|T\| = \inf\{K : K \geq 0, \|T\ (x)\| \leq K\|x\|\} \;\forall\; x \in N$