UTTARAKHAND OPEN UNIVERSITY, HALDWANI (NAINITAL)
उत्तराखंड मुक्त विश्वविद्यालय, हल्द्वानी (नैनीताल)

M.Sc. Mathematics
ASSIGNMENT-SECOND YEAR

Last Date of Submission: 15 May 2015 जमा करने की अन्तिम तिथिः 15 May 2015

Course Title: Analysis and Advanced Calculus **Course Code: MAT506**

Year: 2014-15 **Maximum Marks : 40**

Section 'A'

Section 'A' contains 08 short answer type questions of 5 marks each. Learners are required to answers 4 questions only. Answers of short answer-type questions must be restricted to 250 words approximately.

1. Show that every normal space is metric spaces.

2. Let T : B $\xrightarrow[Linear\ transformation]{Continous}$ B' , then show that T is an open mapping. B and B' are Banach spaces.
3. Define a convex set. Give an example.
4. Define orthonormla set. Show that an orthonormal set S in a Hilbert space H is complete iff $x \perp x \Rightarrow x = 0, \ \forall\, x \in H$.

5. Prove that every Hilbert space is reflexive.

6. Prove that every scalar multiple of self-adjoint operator is also normal.

7. Let T: H + H , then show that T is singular $\Leftrightarrow \exists$ a non-zero vector x in H s.t. Tx =0. H is a finite dimensional Hilbert space.

8. Define directional derivative. Give an example.

Section 'B'

- **Section 'B' contains 04 long answer-type questions of 10 marks each. Learners are required to answers 02 questions only.**

1. Let f: [a, b] $\rightarrow$ x and g: [a, b] $\rightarrow$ R are continuous and differentiable function s.t.

   $\|D f(t)\| \leq D g(t) \; \forall \; t \in (a, b)$, then

   $\|f(b) - f(a)\| \leq g(b) - g(a)$

2. State and prove Hahn-Banch theorem.
3. State and prove Bessel's inequality in Hilbert space.
4. Show that the set of unitary operators on a Hilbert space H, forms a multiplicative group.