UTTARAKHAND OPEN UNIVERSITY, HALDWANI (NAINITAL)

उत्तराखंड मुक्त विश्वविद्यालय, हल्द्वानी (नैनीताल)

M.A./M.Sc. Mathematics
ASSIGNMENT-FIRST YEAR

Last Date of Submission: 15 May जमा करने की अन्तिम तिथिः 15 मई

Course Title: Advanced Algebra **Course Code: MAT 501**

Year: 2013-14 **Maximum Marks : 40**

Section 'A'
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Section 'A' contains 08 short answer type questions of 5 marks each. Learners are required to answers 4 questions only. Answers of short answer-type questions must be restricted to 250 words approximately.

Briefly discuss the following:

1. Let G be a group. H and K are two subgroup of G such that H and K are normal in G and $H \cap K = \{e\}$
   Then show that HK is the internal direct product of H and K.

2. Let G be a finite group and $a \in G$
   then $0(c[a]) = \frac{0\ (G)}{0\ [N\ (a)]}$

3. Show that composition series is not necessarily unique.

4. Prove that Every Euclidean ring is a principal ideal domain.

5. If $T: M \rightarrow N$ is homomorphism, then T is isomorphism if and only if K (T)= {0}, where M and N are R-module.

6. Let V and V' be vector spaces over the field F then prove that the set Hom (V,V') of all linear transformation of V to V' is a vector space over the field F.

7. Let K be an extension of a field F.
   Then the elements in K which are algebraic over F form a subfield of K.

8. Show that every field of characteristic zero is perfect.

Section 'B'

- **Section 'B' contains 04 long answer-type questions of 10 marks each. Learners are required to answers 02 questions only.**

1. Let $V=R^3$, and $t:V\rightarrow V$ be a linear map, defined by t(x, y, z) = (x+z,-2x+y, -x+2y+z) what is the matrix of t with respect to basis {(1, 0, 1), (-1, 1, 1), (0, 1, 1)}
2. Let A be an n x n matrix over a field F, has n distinct Eigenvalues $\lambda_1, \lambda_2, \ldots \lambda_n$ then there exists an invertible matrix P such that $P^{-1}AP = \text{diag}(\lambda_1, \lambda_2, \ldots \lambda_n)$
3. Prove that u and v are orthogonal in an inner product space V if and only if $\|u-v\|^2 = \|u\|^2 + \|v\|^2$
4. Let a and b be two non-zero elements of a Euclidean ring R such that b is not a unit in R, then show that d (ab) > d (a).