



UTTARAKHAND OPEN UNIVERSITY, HALDWANI (NAINITAL)  
उत्तराखंड मुक्त विश्वविद्यालय, हल्द्वानी(नैनीताल)

**M.A./ M.Sc. Mathematics**  
**ASSIGNMENT- FIRST YEAR**

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*Last Date of Submission:* 15 May जमा करने की अन्तिम तिथि: 15 मई

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**Course Title:** Advanced Algebra

**Course code:** MAT 501

**Year:** 2012-13

**Maximum Marks :** 40

**Section 'A'**

**भाग क**

**Section 'A' contains 08 short answer type questions of 5 marks each. Learners are required to answers 4 questions only. Answers of short answer-type questions must be restricted to 250 words approximately.**

Briefly discuss the following:

1. Let  $G$  be any group,  $a$  is a fixed element in  $G$ . Defined  $f:G \rightarrow G$  by  $f(x)=axa^{-1}$ . Prove that  $f$  is an isomorphism of  $G$  onto  $G$ .
1. If  $R$  be a Euclidian ring. Then any two elements  $a$  and  $b$  in  $R$  have a greatest common divisor  $d$ . Moreover,  $d=\lambda a + \mu b$  for some  $\lambda, \mu \in R$ .
2. If  $R$  is a ring and  $I$  is any left ideal in  $R$ , then  $I$  is a left module over  $R$ .
3. If  $G$  is a solvable group and if  $G_1$  is a homomorphic image of  $G$ , then  $G_1$  is solvable.
4. If  $T$  is a linear transformation on a finite dimensional vector space  $V$  over  $F$  then  $T$  is regular if and only if  $T$  maps  $V$  onto  $V$ .
5. If  $\alpha, \beta \in V$  then  $I(u,v)I \leq I\alpha I I\beta I$ .
6. Show that a square matrix  $A$  is invertible if and only if  $|A| \neq 0$ .
7. Define
  - (i) Composition Series
  - (ii) Nullity of Matrices.

**Section 'B'**

**भाग ख**

**Section 'B' contains 04 long answer-type questions of 10 marks each. Learners are required to answers 02 questions only.**

1. Let  $V$  be a finite dimensional inner product space, then  $V$  has an Orthonormal set as a basis.
2. If  $L$  is a finite extension of  $K$  and if  $K$  is a finite extension of  $F$ , then  $L$  is a finite extension of  $F$ . Moreover  $[L:F]=[L:K][K:F]$ .
3. Show Gaussian integer  $J(i)$  is a Euclidean ring.
4. Every finite abelian group is the direct product of cyclic groups.